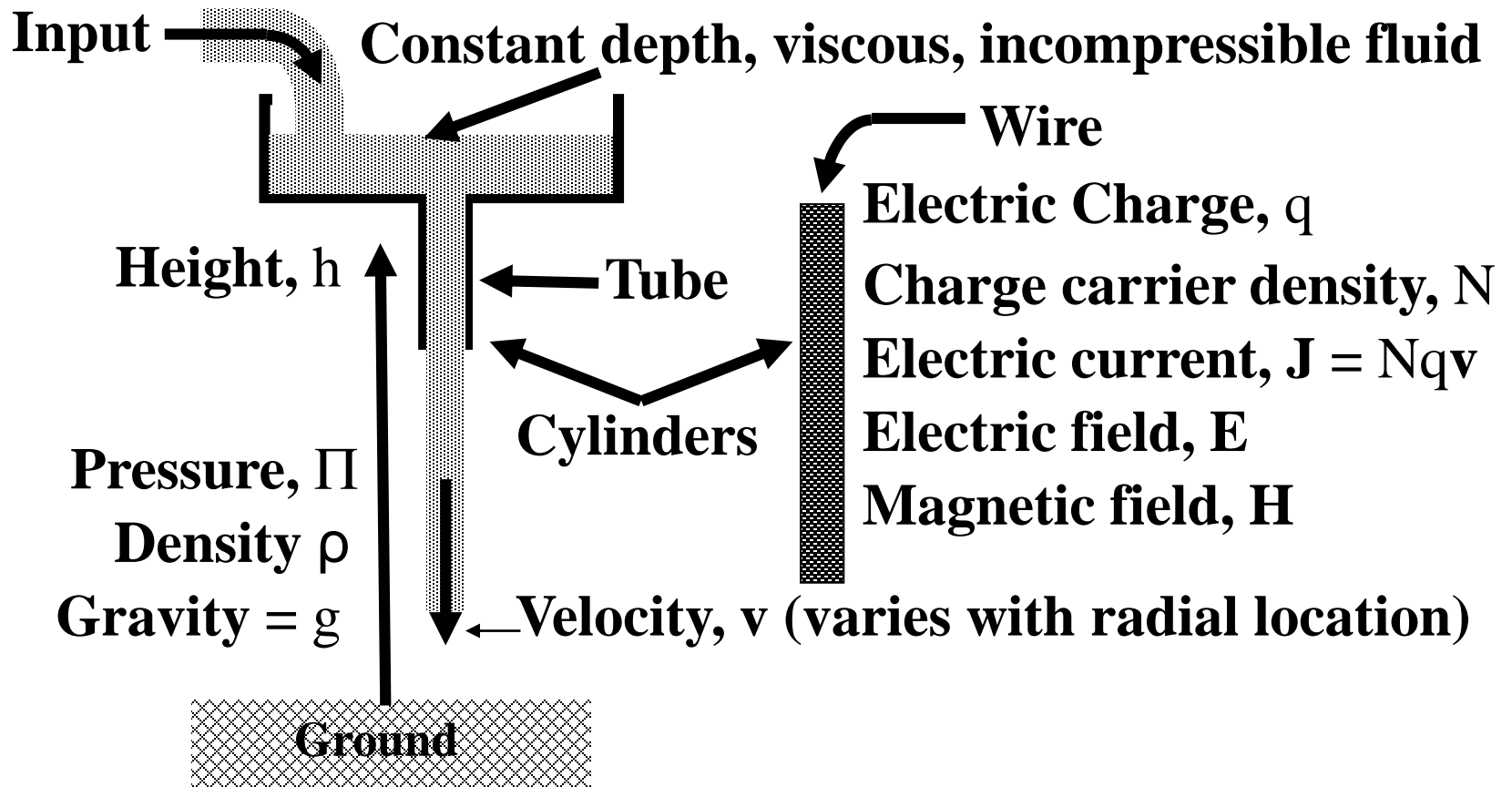**ED05:   2:10-2:20 p.m.   Applying Poynting's Energy Flux to Hydrodynamic Systems**

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By applying Poynting's reasoning to time-independent laminar flows of viscous incompressible fluids we show that the energy flux for such systems can also be described by a Poynting vector. We discuss two examples of the Poynting model of energy flux: the familiar case of a constant electric current flowing through a wire and a new hydrodynamic case of vertical tube which drains a constant depth reservoir which is filled with a viscous fluid. Finally, we present often ignored physical constraints which these systems have to obey and modify Poynting vector so that it can be used, consistently, in order to obtain the actual energy flux for the two systems.

Input → **Constant depth, viscous, incompressible fluid**

Wire

Electric Charge, q

Height, h   **Tube**

Charge carrier density, N

Electric current, J = N q**v**

Cylinders

Electric field, E

Magnetic field, H

Pressure, Π
Density ρ
Gravity = g

Velocity, v (varies with radial location)

Ground

Power input per unit volume = U =
(**Force**/unit volume)**.velocity** = **F.v**

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= (**Force**/unit volume)**.velocity** = **F.v**

Electrical system   (Magnetic field = $\mathbf{B} = \mu_o\mu\mathbf{H}$)

U = **F.v** = **v.** Nq[**E** + (**v**x**B**)] = **J.E**   Nq**v** = **J** = **curl**(**H**)

& because **curl**(**E**) = 0, **E** = **grad**(φ_E) = **grad**(φ)

∴ U = **grad**(φ)**.curl**(**W**) where **W** = **H**

Axisymmetric Hydrodynamic System

F = – **grad**[Π + ρgh] = **grad**(φ_L) and because div(**v**) = 0, **v**

= **curl**(**W**), & ∴ U = **grad**(φ)**.curl**(**W**)

For both systems   U = **grad**(φ)**.curl**(**W**)

Conservation of Energy Equation

$\mathrm{div}(\mathbf{S}) + U = 0$, $\mathbf{S}$ = energy flow per unit area

associated with the flowing materials for the two systems.

Crucial Vector Identity

$$\mathrm{div}[\mathbf{grad}(\varphi)\mathrm{x}\mathbf{W}] = \mathbf{W}\cdot\mathbf{curl}[\mathbf{grad}(\varphi)] - \mathbf{grad}(\varphi)\cdot\mathbf{curl}(\mathbf{W})$$

$$\mathbf{W}\cdot(0) - \mathbf{grad}(\varphi)\cdot\mathbf{curl}(\mathbf{W}) = -U.$$

$$\therefore \quad \mathrm{div}[\mathbf{grad}(\varphi)\mathrm{x}\mathbf{W}] + U = 0 \quad \text{and} \quad \mathrm{div}[\mathbf{S}] + U = 0$$

Tempting [but illogical] Conclusion

$\mathbf{grad}(\varphi)\mathrm{x}\mathbf{W}$ is the _actual_ energy flux $\mathbf{S}$

In EM theory, $\mathrm{div}(\mathbf{S}) = \mathrm{div}[\mathbf{grad}(\varphi)\mathrm{x}\mathbf{W}] = \mathrm{div}(\mathbf{E}\mathrm{x}\mathbf{H})$

The energy flux, $\mathbf{S} = \mathbf{P} = \mathbf{E}\mathrm{x}\mathbf{H}$ is **Poynting's vector**

The generalized Poynting vector is $\mathbf{P}$ where

$$\mathbf{P} = \mathbf{grad}(\varphi)\mathrm{x}\mathbf{W}$$

The Generalized Poynting Vector is P

where P = grad(φ)xW

Direction of P

P is perpendicular to grad(φ) [Because **grad(φ).P** = 0]

Thus, there is no component of P along the symmetry axis

Yet, for the EM model – if there is no axial energy flow then

there is no J; no J means no H and no H means, no ExH

Problem Resolution: Replace P by the correct axial flux, C

C in the two systems, must be solely along the flow axis. Hence,

S must be replaced by P + curl(T) = Cu_z. [u_z = axial unit vector]

div(**C**) + U = 0 = dC/dz + U so that **C** = [− Uz + G(r)]**u**$_z$

Conversion of Poynting's vector, $\mathbf{P}(r)$, into $\mathbf{C}$

$\mathbf{u}_r$, $\mathbf{u}_\theta$ and $\mathbf{u}_z$ = Unit vectors for

cylindrical polar co-ordinates:

$$\mathbf{C} = \mathbf{P}(r) + \mathbf{curl}(\mathbf{T})$$

SOLUTION for T: $\mathbf{T} = T\mathbf{u}_\theta = [P(r)z + (1/r)\, rG(r)dr]\mathbf{u}_\theta$

$$\therefore \mathbf{curl}(\mathbf{T}) = -\,(dT/dz)\mathbf{u}_r + (1/r)[d/dr(rT)]\mathbf{u}_z$$

$$= -\,P(r)\mathbf{u}_r + \{z\,\mathrm{div}[\mathbf{P}(r)] + G(r)\}\mathbf{u}_z.$$

i.e. $\mathbf{curl}(\mathbf{T}) = -\,P(r)\mathbf{u}_r + [-\,Uz + G(r)]\mathbf{u}_z$. Hence, as

required, $\mathbf{P}(r) + \mathbf{curl}(\mathbf{T}) = [-\,Uz + G(r)]\mathbf{u}_z = \mathbf{C}$

$G(r)$ **represents the flux associated with energy**

stored in the moving material of the two systems

Based on the two examples treated in this talk, it is clear that the discussions of energy fluxes, in many contemporary textbooks, are logically flawed and need to be revised.

Poynting's vector is a <u>particular</u> solution of the energy conservation equation which does correctly predict the heat production rate, U.

However, Poynting's vector, P, and the correct energy flux are perpendicular to each other. Furthermore, the axial energy flux $G(r)$ **which is needed to generate H [and, hence, P (= ExH)] is, invariably, ignored.**

To find the correct energy flux $\mathbf{C}$

1) Calculate, U, the power deposited per unit volume per second using $U = \mathbf{F.v}$ where $\mathbf{F}$ is the force per unit volume which produces, $\mathbf{v}$, the velocity of the flowing matter.

2) Avoid using mathematical identities to convert U into the divergence of a vector.

3) Solve the equation, $\text{div}(\mathbf{C}) + U = 0$ with $\mathbf{C}$ directed along $\mathbf{v}$. [i.e. Energy conservation.]

4) In the solution, include the energy flux which occurs because the flowing substance retains stored energy as it moves through any particular system [thereby creating the term, $G(r)\mathbf{u_z}$].